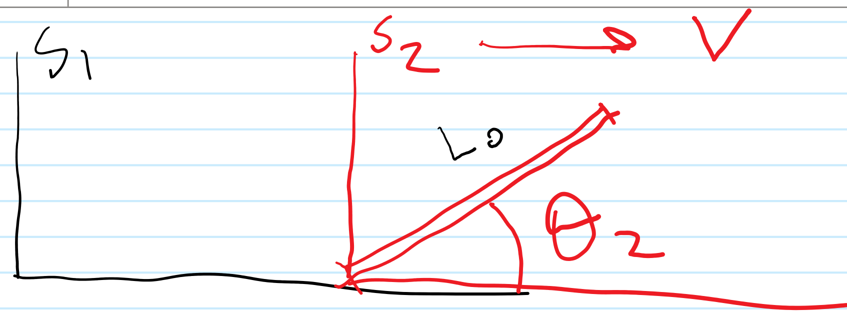
HW Ch. 4

4-6	Given the metersick in example 4.3 at some angle in frame 2, which moves with respect to frame 1, find general relations between the angles θ_1 θ_2 and γ Also find the new length in terms of θ_2 and γ



(1) $\tan\theta_2 = \dfrac{L_{y_0}}{L_{x_0}}$ (2) $\tan\theta_1 = \dfrac{L_y}{L_x}$

(3) $L_x = \dfrac{L_{x_0}}{\gamma}$ & $L_y = L_{y_0}$

so

$$L_x \tan\theta_1 = L_{x_0}\tan\theta_2$$

$$\tan\theta_2 = \frac{\tan\theta}{\gamma}$$

Done relating θ_1 θ_2

$$L = \sqrt{L_x^2 + L_y^2}$$
$$= \sqrt{\left(\frac{L_{x_0}}{\gamma}\right)^2 + L_y^2}$$
$$= L_0\sqrt{\frac{\cos^2\theta_2}{\gamma^2} + \sin^2\theta_2}$$

$L_{oy} = L_0 \sin\theta_2$

$L_{ox} = L_0 \cos\theta_2$

Done - have $L(\theta_2\ \gamma)$

4-7

Same as t_1'

Positions x'_1 and x_1 are measured simultaneously in frame 1 so that $t'_1=t_1$.
Show that the corresponding times in frame 2 are not simultaneous.

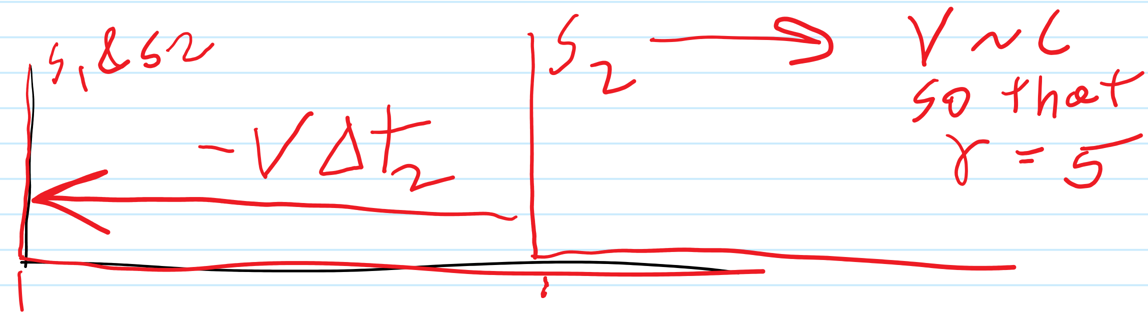
$$x_2 = \gamma(x_1 - vt_1) \qquad x_2' = \gamma(x_1' - vt_1)$$

$$t_2 = \gamma\left(t_1 - \frac{\beta}{c}x_1\right) \qquad t_2' = \gamma\left(t_1 - \frac{\beta}{c}x_1'\right)$$

The times in frame 2 are clearly not equal . The t terms cancel looking at these, but the x_1' and x_1 are given as different, so frame 2 times must be different.

Note that x2' -x2 does not define a proper length of any kind--since not measured at the same time.

Problem
4-9
A particle with lifetime (in its own rest frame) moves by you. You see the particle move with a speed so that γ =5 and the particle observes you falling back behind it by 1200 meters. How far do you see the particle move. The lifetime of the particle is 4.00μs in the rest frame



at $t = 0$ $\qquad t_2 = 4.00\ \mu s$

$$X_2 = -1200\,m$$

NOTE: THE AUTHOR HAS DONE SOME ROUNDING WITH "v". We are given that 1200 m passes in 4.00microseconds. This gives "c" as the speed. If we solve from gamma =5, we get a SLIGHTLY slower speed. So, I should adjust the rounding on that 1200 meters. But it is small. I will use the given 1200m, the 4.00microseconds, and gamma directly without calculating v.

Observer in frame 2 (Colbert) sees you move back by v*Δt

So, Colbert sees you fall back by 1200m at a speed we agree on (rounded almost c to give gamma=5). This takes 4 microseconds according to Colbert.

The two observers--you and Colbert, may disagree about when and how far.
BUT---I see you move back at "v" and you see me move forward at v (you can try velocity transforms here if you wish).

The time you see on your watch as mine hits 4.00 μs

$$T = \gamma T_0 \quad (\text{or } t_2')$$

$$(= 20.0\ \mu s$$

So observer 1 sees for distance traveled as:

$$\Delta x_1 = v\,\Delta t_1$$

$$= v\,(20\,\mu s)$$

With the speed the same as before (that rounded speed of light) and the time 5 times as much due to Gamma, we simply have 5 times the distance. We must since v is the same.

$$= 6000\ m$$

You hold your meterstick still in your frame. Colbert moved 6000m in 20 microsec, at the agreed upon speed v as measured by you.

4-16

A pendulum swings on a rocketship moving 0.600c with respect to YOU/US. The period at rest is 2.00s. What do we see?

$$T_{me} = \gamma T_0$$

$$= \frac{1}{\sqrt{1-0.6^2}} T_0$$

$$= 2.5 \text{ s}$$

The time passing on my watch is longer than the time I see passing on the moving clock.

4-18

An Ω- particle moves at speed so that γ=9.00.   The lifetime I see for the moving particle is 7.4E-18 s.

a) How far do I see the particle move
b) What is the rest lifetime
c) How far does the particle see you move back during its lifetime

$\gamma \rightarrow$

$V = 2.963 \times 10^8\ m/s$

$$\Delta X_{me} = V T_{me}$$
$$= 0.218\ m$$

The rest lifetime will be the shortest---so we always see moving clocks run slow--that means they take longer.

$$T_0 = \frac{T_{me}}{\gamma} = 8.22 \times 10^{-11}\ s$$

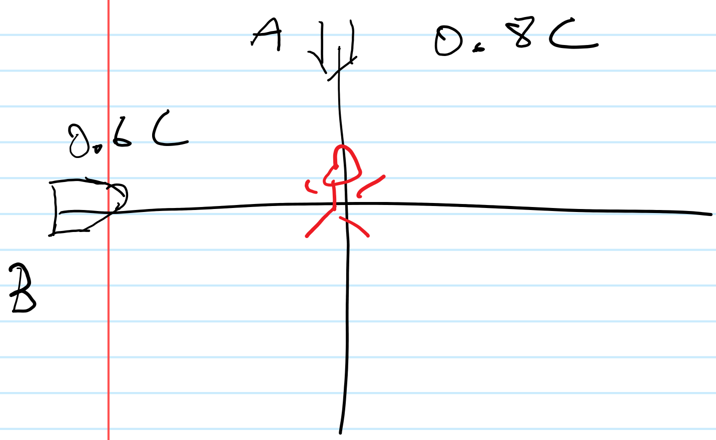
The particle really just sees me moving backward (like problem 4-9) so we work with v which is the same for either observer (particle or me).

$$\Delta X_{\text{particle sees me}} = V T_0$$
$$= 0.02436\ m$$

Gamma came in once due to time dilation calculating the Proper time.

Or we could Treat result from part a as a length
And contract it by the same amount.
Either way.

4-32 Draw this



The clock in rocket A ticks once per second within rocket A (at rest).   How much time does observer in rocket B observe passes in rocket B as clock A ticks once?

We are given velocities in frame of "Origin man".   We need to transform to get the velocity components of rocket A according to observer in B. Then we can get gamma, then clock.   FIRST VELOCITIES.

Frame 1 is origin man, Frame 2 is rocket B, and the object being observed is rocket A.

$$V_{1x} = 0 \qquad V_{1y} = -0.8C \qquad V_z = 0$$

$$V_{2x} = \frac{\cancel{V_{1x}} - V}{1 - 0.6\cancel{V_{1x}}/C} \qquad V = 0.6C \qquad \gamma = 1.25$$

$$= -V$$

$$V_{2y} = \frac{-0.8C}{\gamma\left(1 - 0.6V_{1x}/C\right)} = -0.64C$$

$$V_{2z} = 0$$

4-32 continued

Now use Pythagorean theorem, put together, find v and gamma

$$V_{21} = \sqrt{V_{2x}^2 + V_{2y}^2 + V_{2z}^2} = 0.877c$$

$$\gamma_{BA} = \frac{1}{\sqrt{1-(0.877)^2}} = 2.083$$

$$\overline{T}_B = 2.083 \text{ s}$$

4-36

The transverse Doppler effect is observed---so a source is emitting and passing you by at an angle (as you point toward the emitter) at 90º.

The wave is emitted in all directions, I've just shown where the source is when emitting peaks/troughs etc.

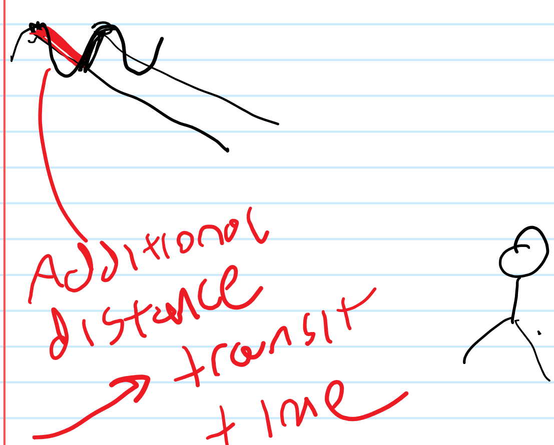
When at 90 degrees, the Doppler shift formula reduces to:

$$\frac{1}{T} = \frac{1}{T_0}\,\frac{1}{\gamma} \qquad T = \gamma T_0$$

The transit time effect is gone at 90 degrees, the entire remaining effect is due to time dilation.

Why is this result not true for all angles?

For any other angle, the additional effect of transit time also impacts the measurement of observed (apparent) frequency.



Colbert Cyop

Mr. Spock is driving a Delorean shaped space ship at high speeds, and passes by a bewildered Ape named Dr. Zaius. Spock is moving at 0.8000c according to Dr Zaius.

a) Dr. Zaius reads 1.00s on Mr. Spock's moving watch, what time is shown on the clock at rest with Dr. Zaius.sS
b) The Flash now passes Dr. Zaius at a speed of 0.9000c. What speed does Spock observe the Flash moving?
c) All three of our scientists have been tagged with an Arduino constructed blinky light emitting light at 650nm. Determine the frequency of the light in the rest frame, and frequencies as observed by Dr. Zaius after each Flash and Spock have passed by some time ago.

a) $$\gamma_{sp} = \frac{1}{\sqrt{1-(0.8)^2}} = 1.667$$

$$T = \gamma T_0 = 1.667 \text{ s}$$

b) object = flash $V_{1x} = 0.9c$

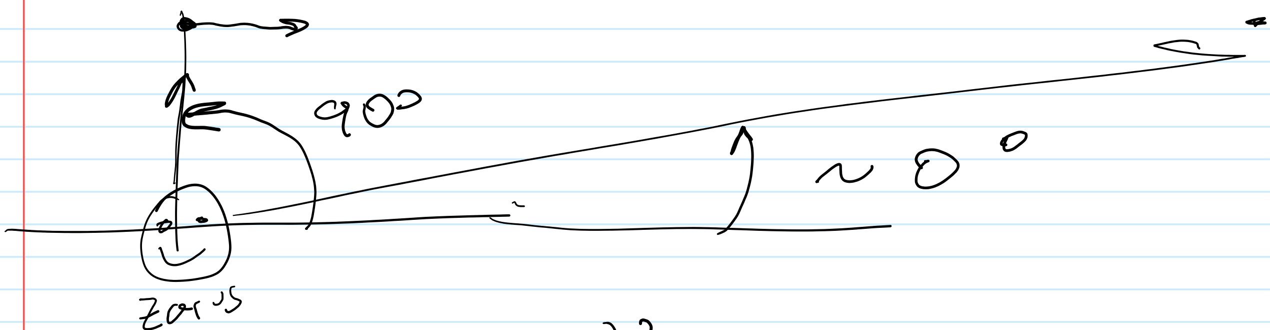
frame 1 = Zaius

frame 2 = Spock $V = 0.8c$

$$V_{2x} = \frac{V_{1x} - V}{1 - \frac{\beta}{c} V_{1x}}$$

$$= \frac{0.9c - 0.8c}{1 - 0.8 \times 0.9\cancel{c}/\cancel{c}} = \underline{0.357c}$$

c) $\nu_0 = \frac{c}{650\,nm} = 4.615 \times 10^{14}$ HZ



$$\nu = \frac{\nu_0}{\gamma\left(1 + \frac{v}{c}\cos\theta\right)}$$

$$\nu_{90\,flash} = \frac{\nu_0}{2.29\,(1+0)} = 2.015 \times 10^{14}\ HZ$$

$$\nu_{90\,spaceL} = \frac{\nu_0}{1.667} = 2.768 \times 10^{14}\ HZ$$

$$\nu_{0\,fL} = \frac{\nu_0}{2.29\,(1+0.9)} = 1.061 \times 10^{14}\ HZ$$

$$\nu_{0\,SP} = \frac{\nu_0}{1.667\,(1+0.8)} = 1.538 \times 10^{14}\ HZ$$